APPLICATION OF FIBONACCI EXP-FUNCTION METHOD FOR SOLVING THE PARTIAL DIFFERENTIAL EQUATIONS

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Abstract. In this work, we establish the exact solutions to the longitudinal wave motion equation in a nonlinear magneto-electro-elastic circular rod and Burgers`equations. The Fibonacci exp-function method were used to construct solitary wave solutions of nonlinear evolution equations. The Fibonacci exp-function method presents a wider applicability for handling nonlinear wave equations. It is shown that the Fibonacci exp-function methods, with the help of symbolic computation, provide a straightforward and powerful mathematical tool for solving nonlinear evolution equations in mathematical physics.

Key words: solitary wave solutions, Fibonacci exp-function method, Burgers` equation, longitudinal wave motion equation.

Introduction

In the recent years, the investigation of the traveling wave solutions for nonlinear partial differential equations plays an important role in the study of nonlinear physical phenomena. During the past decades, both mathematicians and physicists have devoted considerable effort to the study of exact and numerical solutions of the nonlinear ordinary or partial differential equations corresponding to the nonlinear problems. Many powerful methods have been presented. For instance, the Hirota's bilinear method [11], the inverse scattering transform [1], F-expansion method [19], sine-cosine method [21], homotopy perturbation method [2, 3], homotopy analysis method [4], variational iteration method [3, 10], tanh-function method [8], generalized tanh-coth method [13, 14, 17], Bäcklund transformation [18], Exp-function method [5, 6, 7, 12 15], $(\frac{G'}{G})$-expansion method [9] and so on. The Burgers equation [16]

$$u_t + uu_x = u_{xx}, \tag{1}$$

is a nonlinear partial differential equation of second order which appears in various areas of applied mathematics, such as modeling of fluid dynamics, turbulence, boundary layer behavior, shock wave formation, and traffic flow. Based on the constitutive relation for transversely isotropic piezoelectric and piezo-magnetic materials, combined with the differential equations of motion, Xue et al. [22] have derived a longitudinal wave motion equation in a MEE circular rod of the form,

$$u_{tt} - c_0^2 u_{xx} - \left(\frac{c_0^2}{2} u^2 + N u_{tt}\right)_{xx} = 0. \tag{2}$$

where c_0 is the linear longitudinal wave velocity for a MEE circular rod and N is the dispersion parameter, both depending on the material properties as well as the geometry of the rod. Equation (2) is a nonlinear wave equation with dispersion caused by the transverse Poisson's effect. Here, we use of an effective method namely the Fibonacci exp-function method for constructing a range of exact solutions for the following ordinary partial differential equations that in this paper we developed solutions as well. In this paper, we put forth the new approach of Fibonacci exp-function method to construct exact travelling wave solutions including solitons, kink, periodic and rational solutions to the longitudinal wave motion equation in a nonlinear magneto-electro-elastic circular rod and Burger equations. The purpose of this paper is to obtain exact solutions of the longitudinal wave motion equation in a nonlinear magneto-electro-elastic circular rod and Burger equations. and to determine the accuracy of the Fibonacci exp-function method in solving these kind of problems. The article is organized as follows: In Section 2, first we briefly give the step of the method and apply this method to solve the nonlinear partial differential equations. In Section 3, the application of the Fibonacci exp-function method to the longitudinal wave motion equation in a nonlinear magneto-electro-elastic circular rod and Burger equations will be introduced briefly. Also a conclusion is given in Section 4. Finally some references are given at the end of this paper.

1. Fibonacci exp-function method

The symmetrical Fibonacci sine (sFs) and the symmetrical Fibonacci cosine (cFs) are defined as follows

$$\mathrm{sFs}(x) = \frac{a^x - a^{-x}}{\sqrt{5}}, \tag{3}$$

$$\mathrm{cFs}(x) = \frac{a^x + a^{-x}}{\sqrt{5}}, \tag{4}$$

and they have the properties

$$a^x = \frac{\sqrt{5}}{2}(\mathrm{sFs}(x) + \mathrm{cFs}(x)), \tag{5}$$

$$a^{-x} = \frac{\sqrt{5}}{2}(\mathrm{cFs}(x) - \mathrm{sFs}(x)), \tag{6}$$

$$(\mathrm{cFs}(x))^2 - (\mathrm{sFs}(x))^2 = \frac{4}{5}, \tag{7}$$

$$(\mathrm{cFs}(x))' = \ln a(\mathrm{sFs}(x)), \quad 0.5\mathrm{cm}(\mathrm{sFs}(x))' = \ln a(\mathrm{cFs}(x)). \tag{8}$$

Obviously, when $a = e$,

$$sFs(x) = \frac{2\sinh(x)}{\sqrt{5}}, \quad cFs(x) = \frac{2\cosh(x)}{\sqrt{5}}. \tag{9}$$

We first consider the nonlinear equation of the form

$$\mathcal{N}(u, u_t, u_x, u_{xx}, u_{tt}, u_{tx}, \dots) = 0, \tag{10}$$

and introduce the transformation as

$$u(x,t) = u(\eta), \quad \eta = kx + \omega t, \tag{11}$$

where c is constant to be determined later, therefore Eq. (10) reduced to ODE as follow

$$\mathcal{M}(u, \omega u', ku', k^2 u'', \dots) = 0. \tag{12}$$

The Fibonacci EFM is based on the assumption that travelling wave solutions can be expressed in the following form

$$u(\eta) = \frac{A_0 + A_1 a^{\eta} + A_2 a^{2\eta} + \dots + A_m a^{m\eta}}{B_0 + B_1 a^{\eta} + B_2 a^{2\eta} + \dots + B_m a^{m\eta}}. \tag{13}$$

where A_0, $A_k (k = 1,2, \dots, m)$, B_0, $B_k (k = 1,2, \dots, m)$ λ and μ are constants to be determined later, $A_m \neq 0$, $B_m \neq 0$, but the degree of which is generally equal to or less than $m - 1$, the positive integer m can be determined by considering the homogeneous balance between the highest order derivatives and nonlinear terms appearing in Eq. (10).

2. Application of the Fibonacci EFM to Burger equation

The Burger equation which has been extensively investigated. The nonlinear Burger equation is a nonlinear partial differential equation of second order which appears in various areas of applied mathematics, such as modeling of fluid dynamics, turbulence, boundary layer behavior, shock wave formation, and traffic flow as follows

$$u_t + uu_x = u_{xx}. \tag{14}$$

In this section the Fibonacci EFM will be applied to handle Burger equation. On substituting the transformation rule $\xi = x - ct$ into Eq. (14) we obtain

$$-cu' + uu' - u'' = 0, \tag{15}$$

where by integrating Eq. (15) with respect to ξ, we obtain

$$-cu + \frac{1}{2}u^2 - u' = 0. \tag{16}$$

Using (13), for sake of simplicity let $m = 2$, $u(\xi)$ is expressed in rational expansion form

$$u(\xi) = \frac{A_0 + A_1 a^{\xi} + A_2 a^{2\xi}}{B_0 + B_1 a^{\xi} + B_2 a^{2\xi}}. \tag{17}$$

Substituting (17) into Eq. (16) and equating the coefficients of all powers of a to zero yields a set of algebraic equations for $A_0, A_1, A_2, B_0, B_1, B_2$ and c. Solving, with the help of mathematica programm, the system of algebraic equations, we obtain six sets of solutions as follows:

(I) First set:

$$A_0 = 0,\quad A_1 = 2\ln(a)B_1,\quad A_2 = 0,\quad B_0 = 0,\quad B_1 = B_1,\quad B_2 = B_2,\quad c = \ln(a). \quad (18)$$

We, therefore, obtain the following solution of (14)

$$u_1(x,t) = \frac{2\ln(a)B_1 a^{x-\ln(a)t}}{B_1 a^{x-\ln(a)t} + B_2 a^{2x-2\ln(a)t}}. \quad (19)$$

Remark 1. Our aim in this remark is to show that the Fibonacci Expa-function method could be used to determine traveling wave solutions in the form of symmetrical hyperbolic Fibonacci function. This can be easily obtained by selecting specific values for the parameters that present in the solutions as shown below. Now if we assume $k = B_2/B_1$, solution (19) can be transformed into the following form

$$u_1(x,t) = \frac{2\ln(a)}{1 + \frac{k\sqrt{5}}{2}[sFs(x-\ln(a)t) + cFs(x-\ln(a)t)]}, \quad (20)$$

where a and k are free parameters.

(II) Second set:

$$A_0 = 4\ln(a)B_0 A_1 = 0,\quad A_2 = 0,\quad B_0 = B_0,\quad B_1 = 0,\quad B_2 = B_2,\quad c = 2\ln(a). \quad (21)$$

We, therefore, obtain the following solution of (14)

$$u_2(x,t) = \frac{4\ln(a)B_0}{B_0 + B_2 a^{2x-4\ln(a)t}}. \quad (22)$$

Remark 2. Now if we assume $k = B_2/B_0$, solution (22) can be transformed into the following form

$$u_2(x,t) = \frac{4\ln(a)}{1 + \frac{5k}{4}[sFs(x-2\ln(a)t) + cFs(x-2\ln(a)t)]^2}, \quad (23)$$

where a and k are free parameters.

(III) Third set:

$$A_0 = 2\ln(a)B_0,\quad A_1 = A_1 A_2 = 0,\quad B_0 = B_0,\quad B_1 = B_1,\quad B_2 = -\frac{A_1(-2\ln(a)B_1 + A_1)}{4B_0\ln(a)^2},\quad c = \ln(a). \quad (24)$$

We, therefore, obtain the following solution of (14)

$$u_3(x,t)=\frac{2\ln(a)B_0+A_1a^{x-\ln(a)t}}{B_0+B_1a^{x-\ln(a)t}-\frac{A_1(-2\ln(a)B_1+A_1)}{4B_0\ln(a)^2}a^{2x-2\ln(a)t}}, \tag{25}$$

where a, B_0, A_1 and B_1 are free parameters.

(IV) Forth set:

$$A_0=0A_1=0,\ \ A_2=-4\ln(a)B_2,\ \ B_0=B_0,\ \ B_1=0,\ \ B_2=B_2,\ \ c=-2\ln(a). \tag{26}$$

We, therefore, obtain the following solution of (14)

$$u_4(x,t)=-\frac{4\ln(a)B_2a^{2x+4\ln(a)t}}{B_0+B_2a^{2x+4\ln(a)t}}.\quad 3cm \tag{27}$$

Remark 3. Now if we assume $k=B_0/B_2$, solution (27) can be transformed into the following form

$$u_4(x,t)=-\frac{4\ln(a)}{1+\frac{2k}{5}[sFs(x+2\ln(a)t)+cFs(x+2\ln(a)t)]^{-2}}, \tag{28}$$

where a and k are free parameters.

(V) Fifth set:

$$A_0=0,\ \ A_1=0,\ \ A_2=-2\ln(a)B_2,\ \ B_0=0,\ \ B_1=B_1,\ \ B_2=B_2,\ \ c=-\ln(a). \tag{29}$$

We, therefore, obtain the following solution of (14)

$$u_5(x,t)=-\frac{2\ln(a)B_2a^{2x+2\ln(a)t}}{B_1a^{x+\ln(a)t}+B_2a^{2x+2\ln(a)t}}. \tag{30}$$

Remark 4. Now if we assume $k=B_1/B_2$, solution (30) can be transformed into the following form

$$u_5(x,t)=-\frac{2\ln(a)}{1+\frac{2k}{\sqrt{5}}[sFs(x+\ln(a)t)+cFs(x+\ln(a)t)]^{-1}}, \tag{31}$$

where a and k are free parameters.

(VI) sixth set:

$$A_0=0,\ \ A_1=A_1,\ \ A_2=\frac{A_1(2\ln(a)B_1+A_1)}{2\ln(a)B_0},\ \ B_0=B_0,\ \ B_1=B_1,\ \ B_2=-\frac{A_1(2\ln(a)B_1+A_1)}{4\ln(a)^2B_0}. \tag{32}$$

We, therefore, obtain the following solution of (14)

$$c=-\ln(a),\ \ u_6(x,t)=\frac{A_1a^{x+\ln(a)t}+\frac{A_1(2\ln(a)B_1+A_1)}{2\ln(a)B_0}a^{2x+2\ln(a)t}}{B_0+B_1a^{x+\ln(a)t}-\frac{A_1(2\ln(a)B_1+A_1)}{4\ln(a)^2B_0}a^{2x+2\ln(a)t}}. \tag{33}$$

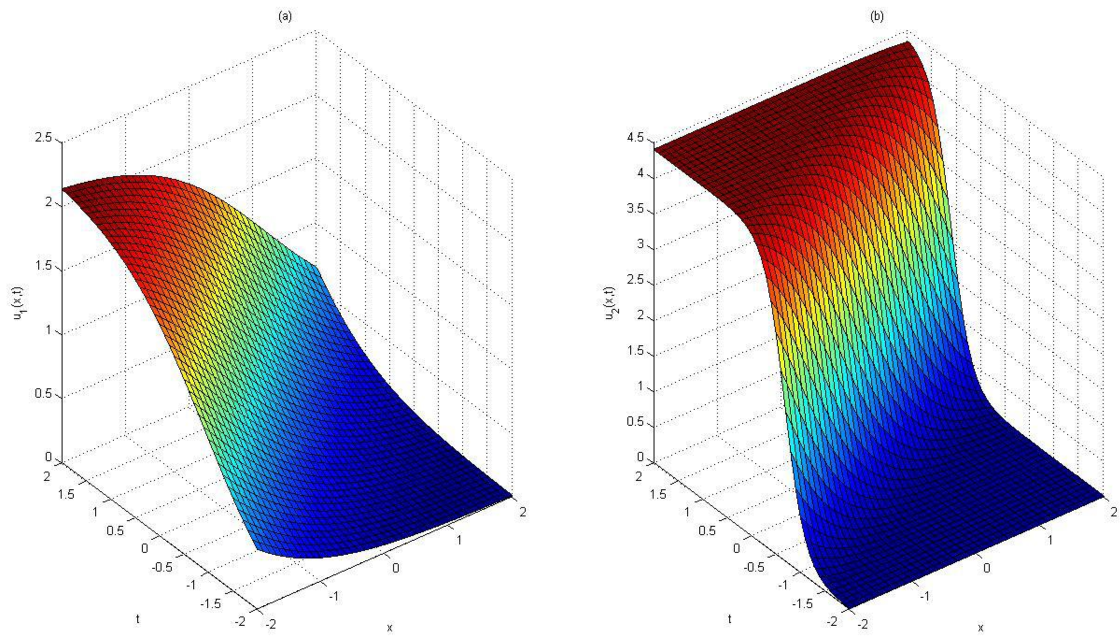

Figure 1: The surface of the exact solutions of Burgur equation (a) (19) u_1 when $a = 3, B_0 = 1$ and $B_2 = 3$ and (b) (22) u_2 when $a = 3, B_1 = 1$ and $B_2 = 3$.

3. Application of the Fibonacci EFM to longitudinal wave motion equation

Based on the constitutive relation for transversely isotropic piezoelectric and piezo-magnetic materials, combined with the differential equations of motion, Xue et al. [22] have derived a longitudinal wave motion equation in a MEE circular rod of the form,

$$\mathrm{u_{tt}} - \mathrm{c_0^2 u_{xx}} - \left(\frac{\mathrm{c_0^2}}{2}\mathrm{u}^2 + \mathrm{Nu_{tt}}\right)_{\mathrm{xx}} = 0. \tag{34}$$

In this section the Fibonacci EFM will be applied to handle Burger equation. On substituting the transformation rule $\xi = \mathrm{k(x} - \omega \mathrm{t)}$ into Eq. (34) we obtain

$$\mathrm{k}^2\omega^2\mathrm{u}'' - \mathrm{k}^2\mathrm{c_0^2} - \mathrm{k}^2\left(\frac{\mathrm{c_0^2}}{2}\mathrm{u}^2 + \mathrm{Nk}^2\omega^2\mathrm{u}''\right) = 0. \tag{35}$$

By integrating Eq. (35) twice with respect to ξ, and neglecting the constant of integration, we obtain

$$\mathrm{u}'' + \frac{\mathrm{c_0^2} - \omega^2}{\mathrm{Nk}^2\omega^2}\mathrm{u} + \frac{\mathrm{c_0^2}}{2\mathrm{Nk}^2\omega^2}\mathrm{u}^2 = 0. \tag{36}$$

Using (13), for sake of simplicity let $m = 2$, $u(\xi)$ is expressed in rational expansion form

$$u(\xi) = \frac{A_0 + A_1 a^{\xi} + A_2 a^{2\xi}}{B_0 + B_1 a^{\xi} + B_2 a^{2\xi}}. \qquad (37)$$

Substituting (17) into Eq. (16) and equating the coefficients of all powers of a to zero yields a set of algebraic equations for $A_0, A_1, A_2, B_0, B_1, B_2$ and c. Solving, with the help of mathematica programm, the system of algebraic equations, we obtain six sets of solutions as follows:

(I) First set:

$$A_0 = 0, A_1 = \frac{6(\omega^2 - c_0^2)B_1}{c_0^2}, \quad A_2 = 0, \quad B_0 = B_0, \quad B_1 = B_1, \quad B_2 = \frac{B_1^2}{4B_0}, \qquad (38)$$

$$\omega = \omega, \quad k = \frac{1}{\omega \ln(a)}\sqrt{\frac{\omega^2 - c_0^2}{N}}.$$

We, therefore, obtain the following solution of (14)

$$u_1(x,t) = \frac{\frac{6(\omega^2 - c_0^2)B_1}{c_0^2} a^{\frac{1}{\omega \ln(a)}\sqrt{\frac{\omega^2 - c_0^2}{N}}(x - \omega t)}}{B_0 + B_1 a^{\frac{1}{\omega \ln(a)}\sqrt{\frac{\omega^2 - c_0^2}{N}}(x - \omega t)} + \frac{B_1^2}{4B_0} a^{\frac{1}{\omega \ln(a)}\sqrt{\frac{\omega^2 - c_0^2}{N}}(2x - 2\omega t)}} = \qquad (39)$$

$$\frac{\frac{24(\omega^2 - c_0^2)B_0 B_1}{c_0^2} a^{\frac{1}{\omega \ln(a)}\sqrt{\frac{\omega^2 - c_0^2}{N}}(x - \omega t)}}{\left(2B_0 + B_1 a^{\frac{1}{\omega \ln(a)}\sqrt{\frac{\omega^2 - c_0^2}{N}}(x - \omega t)}\right)^2}.$$

Remark 1. Our aim in this remark is to show that the Fibonacci Expa-function method could be used to determine traveling wave solutions in the form of symmetrical hyperbolic Fibonacci function. This can be easily obtained by selecting specific values for the parameters that present in the solutions as shown below. Now if we assume $2B_0/B_1 = 1$, solution (39) can be transformed into the following form

$$u_1(x,t) = \frac{6(\omega^2 - c_0^2)}{c_0^2} \frac{\sqrt{5}\left[sFs\left(\frac{1}{\omega \ln(a)}\sqrt{\frac{\omega^2 - c_0^2}{N}}(x - \omega t)\right) + cFs\left(\frac{1}{\omega \ln(a)}\sqrt{\frac{\omega^2 - c_0^2}{N}}(x - \omega t)\right)\right]}{\left(1 + \frac{\sqrt{5}}{2}\left[sFs\left(\frac{1}{\omega \ln(a)}\sqrt{\frac{\omega^2 - c_0^2}{N}}(x - \omega t)\right) + cFs\left(\frac{1}{\omega \ln(a)}\sqrt{\frac{\omega^2 - c_0^2}{N}}(x - \omega t)\right)\right]\right)^2}, \qquad (40)$$

where a and ω are free parameters.

(II) Second set:

$$A_0=\frac{2B_0(\omega^2-c_0^2)}{c_0^2},\quad A_1=-\frac{4B_1(\omega^2-c_0^2)}{c_0^2},\quad A_2=\frac{B_1^2(\omega^2-c_0^2)}{2B_0c_0^2},\quad B_0=B_0,\quad B_1=B_1, \tag{41}$$

$$B_2=\frac{B_1^2}{4B_0},\quad \omega=\omega,\quad k=\frac{1}{\omega\ln(a)}\sqrt{\frac{c_0^2-\omega^2}{N}}.$$

We, therefore, obtain the following solution of (14)

$$u_2(x,t)=\frac{\frac{2B_0(\omega^2-c_0^2)}{c_0^2}-\frac{4B_1(\omega^2-c_0^2)}{c_0^2}a^{\frac{1}{\omega\ln(a)}\sqrt{\frac{c_0^2-\omega^2}{N}}(x-\omega t)}+\frac{B_1^2(\omega^2-c_0^2)}{2B_0c_0^2}a^{\frac{1}{\omega\ln(a)}\sqrt{\frac{\omega^2-c_0^2}{N}}(x-\omega t)}}{B_0+B_1a^{\frac{1}{\omega\ln(a)}\sqrt{\frac{c_0^2-\omega^2}{N}}(x-\omega t)}+\frac{B_1^2}{4B_0}a^{\frac{1}{\omega\ln(a)}\sqrt{\frac{c_0^2-\omega^2}{N}}(2x-2\omega t)}}= \tag{42}$$

$$\frac{2(\omega^2-c_0^2)}{c_0^2}\frac{\left(2B_0-B_1a^{\frac{1}{\omega\ln(a)}\sqrt{\frac{c_0^2-\omega^2}{N}}(x-\omega t)}\right)^2-4B_0B_1a^{\frac{1}{\omega\ln(a)}\sqrt{\frac{c_0^2-\omega^2}{N}}(x-\omega t)}}{\left(2B_0+B_1a^{\frac{1}{\omega\ln(a)}\sqrt{\frac{c_0^2-\omega^2}{N}}(x-\omega t)}\right)^2}.$$

Remark 2. Now if we assume $2B_0/B_1=1$ solution (42) can be transformed into the following form

$$u_2(x,t)=\frac{2(\omega^2-c_0^2)}{c_0^2}\left\{\frac{\left(1-\frac{\sqrt{5}}{2}\left[sFs\left(\frac{1}{\omega\ln(a)}\sqrt{\frac{\omega^2-c_0^2}{N}}(x-\omega t)\right)+cFs\left(\frac{1}{\omega\ln(a)}\sqrt{\frac{\omega^2-c_0^2}{N}}(x-\omega t)\right)\right]\right)^2}{\left(1+\frac{\sqrt{5}}{2}\left[sFs\left(\frac{1}{\omega\ln(a)}\sqrt{\frac{\omega^2-c_0^2}{N}}(x-\omega t)\right)+cFs\left(\frac{1}{\omega\ln(a)}\sqrt{\frac{\omega^2-c_0^2}{N}}(x-\omega t)\right)\right]\right)^2}\right. \tag{43}$$

$$\left.-\frac{\sqrt{5}\left[sFs\left(\frac{1}{\omega\ln(a)}\sqrt{\frac{\omega^2-c_0^2}{N}}(x-\omega t)\right)+cFs\left(\frac{1}{\omega\ln(a)}\sqrt{\frac{\omega^2-c_0^2}{N}}(x-\omega t)\right)\right]}{\left(1+\frac{\sqrt{5}}{2}\left[sFs\left(\frac{1}{\omega\ln(a)}\sqrt{\frac{\omega^2-c_0^2}{N}}(x-\omega t)\right)+cFs\left(\frac{1}{\omega\ln(a)}\sqrt{\frac{\omega^2-c_0^2}{N}}(x-\omega t)\right)\right]\right)^2}\right\},$$

where a and ω are free parameters.

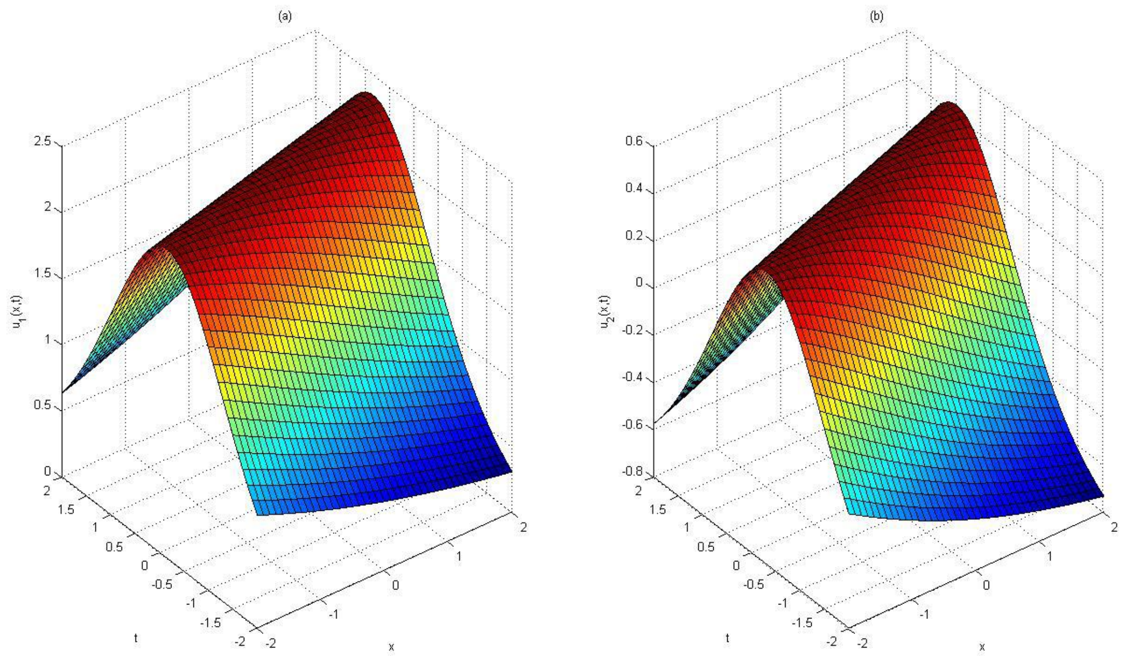

Figure 2: The surface of the exact solutions of longitudinal wave motion equation (a) (39) u_1 when $a = 3, B_0 = 1, B_1 = 3, c_0 = 3$ and $\omega = 4$ and (b) (42) u_2 when $a = 3, B_0 = 1, B_1 = 3, c_0 = 4$ and $\omega = 3$.

In Figures 1-2, the graphical representations of some obtained solutions of aforementioned equations are given.

Conclusion

The Fibonacci exp-function method was successfully used to establish periodic wave and solitary wave solutions. The obtained results complement the useful works of others for this important equations. The Fibonacci exp-function method is a useful method for finding travelling wave solutions of nonlinear evolution equations. The Fibonacci exp-function method is more powerful in searching for exact solutions of NLPDEs. Some of these results are in agreement with the results reported specially by [22]. It can be concluded that the this method is a very powerful and efficient technique in finding exact solutions for wide classes of problems.

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XÜSUSİ DİFFERENSİAL TƏNLİKLƏRİN HƏLLİ ÜÇÜN FİBONAÇÇİ EKSPONENSİAL-FUNKSİYA ÜSULUNUN TƏTBİQİ

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Biz bu işdə uzunasına dalğa hərəkət tənliyinin və Burger tənliklərinin dəqiq həllərini qeyri-xətti maqnit-elektro-elastik dairəvi çubuqda yaradırıq. Əvvəlcə, Fibonaççi eksponensial-funksiya üsulundan qeyri-xətti təkamül tənliklərinin tək dalğa həllərini almaq üçün istifadə edilmişdir. Fibonaççi eksponensial-funksiya üsulu qeyri-xətti dalğa tənliklərini idarə etmək üçün daha geniş tətbiq imkanlarını təqdim edir. Göstərilmişdir ki, Fibonaççi eksponensial-funksiya üsulları simvolik hesablamanın köməyi ilə riyazi fizikada qeyri-xətti təkamül tənliklərinin həlli üçün çox sadə və qüvvətli riyazi cihazı əvəz edir.
Açar sözlər: tək dalğa həllər, Fibonaççi eksponensial-funksiya üsulu, Burger tənliyi; uzunasına dalğa hərəkət tənliyi.

ПРИМЕНЕНИЕ МЕТОДА ЭКСПЕРИМЕНТАЛЬНОЙ ФУНКЦИИ ФИБОНАЧЧИ ДЛЯ РЕШЕНИЯ ЧАСТНЫХ ДИФФЕРЕНЦИАЛЬНЫХ УРАВНЕНИЙ

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В данной работе мы разработали точные решения к уравнению движения продольных волн в нелинейной магнитоэлектроупругой круглой стержне и уравнениям Бюргерса. Метод экспериментальной функции Фибоначчи был использован для построения уединенных волновых решений нелинейных эволюционных уравнений. Метод экспериментальной функции Фибоначчи имеет более широкое применение для обработки нелинейных волновых уравнений. Показано, что методы экспериментальной функции Фибоначчи с помощью символьных вычислений обеспечивают простой и мощный математический инструмент для решения нелинейных эволюционных уравнений в математической физике.
Ключевые слова: уединенные волновые решения, метод экспериментальной функций Фибоначчи, уравнение Бюргерса, уравнение движения продольных волн.

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